J. Fluid Mech. (1983), vol. 132, pp. 247–256
Printed in Great Britain

A technique for the direct measurement of volume flux of a plume

By W. DOUGLAS BAINES
Department of Mechanical Engineering, University of Toronto, Toronto, Canada, M5S 1A4

(Received 15 April 1982 and in revised form 23 September 1982)

It is observed that an interface is established in any open-ended vertical cylinder when a plume flows in the direction of a mean flow through the cylinder. The location of the interface defines the volume flux of the plume because the total discharge in the environment has been entrained at this level. Experiments have been conducted to verify this relation and to define the limits within which the interface is established. It is found that, as the buoyancy in the plume is reduced or the area of the environment is reduced, limits do exist. An application of the interface in industrial-building ventilation is described.

1. Introduction

The measurement of the discharge or volume flux in a conduit can be accomplished readily with a venturi meter producing a pressure difference, but no such device is available for a free shear flow. Velocity must be measured at a large number of points, and the volume flux determined from an integration across the section. This is an accurate procedure, but one which involves a large amount of work, particularly if there is not an axis of symmetry. It was demonstrated by Ricou & Spalding (1961) that the volume flux entrained by a jet can be measured by enclosing it in a cylindrical container and metering the inflow induced to enter. Of course, there must be an opening the size of the jet in the opposite end of the container. This requirement produces an inaccuracy because of the difficulty in defining the edge of a turbulent jet.

It is demonstrated in this paper that, if the free shear flow is buoyant, a simpler technique exists. This is applied to a forced plume, i.e. a jet of buoyant fluid or a pure plume such as produced by a source of heat. The plume is contained in a vertical cylinder of large cross-section with the top and bottom open. The outflow through the bottom is controlled and a small vertical velocity exists throughout this environment. It was shown by Baines & Turner (1969) that a stable stratification develops in an enclosed volume because the velocity of entrainment to the plume is virtually horizontal. A simple application of this concept in §2 demonstrates that an interface develops in the environment at the elevation where the entrainment flux of the plume is equal to the volume flux Q_1 introduced through the top. Thus the measurement of elevation of the interface for various Q_1 provides data for the relationship of the plume volume flux with distance from the source. Section 3 presents a description of laboratory experiments devised to verify and delineate the phenomenon. Data are presented for pure and forced plumes and for a pair of plumes from two closely spaced sources. In §4, an application of the technique to the ventilation of a large metallurgical building is described.

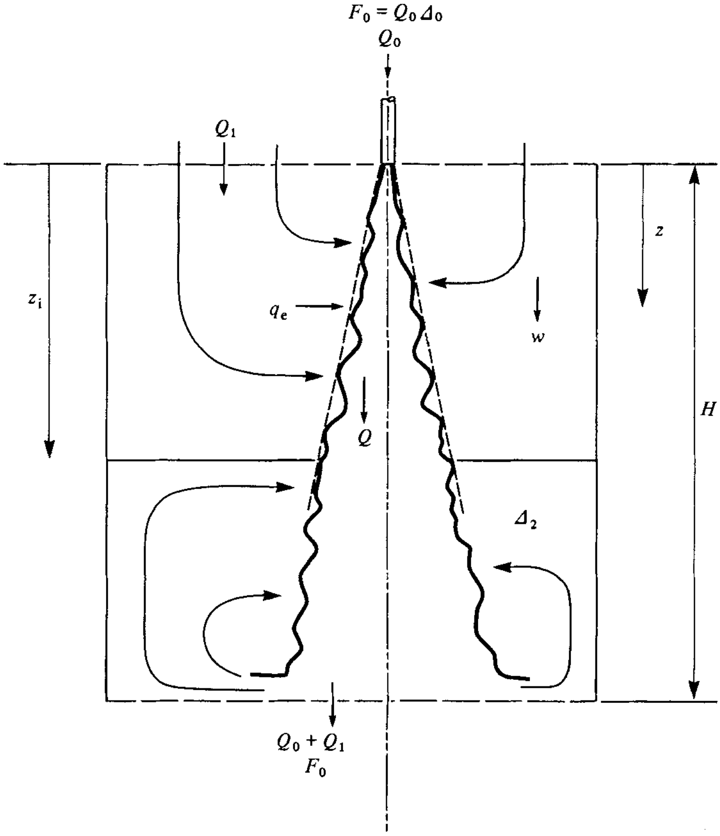

FIGURE 1. Definition sketch of plume in slow vertical flow.

2. Derivation of the technique

Consider the geometry of figure 1 showing a source of volume flux Q_0 and buoyancy flux F_0 near the top of the container. Assuming that buoyancy $\Delta = g\,\Delta\rho/\rho$ is conserved and that molecular diffusion is negligible compared to both convective and turbulent transport, the equation for buoyancy derived by Baines & Turner (1969) is

$$\frac{\partial \Delta}{\partial t} + w\frac{\partial \Delta}{\partial z} = 0. \tag{1}$$

For steady flow, (1) shows that either w or $\partial\Delta/\partial z$ is zero at every point in the environment.

The continuity equation for the container at the inlet and at z is

$$Q_1 + Q_0 = Q + wA, \tag{2}$$

and it is assumed that the entrainment flux

$$q_e = \frac{dQ}{dz} \tag{3}$$

is positive because any characteristic plume velocity is much larger than w. Equation (2) thus shows that there must be an elevation z_i for which w is zero and

$$Q_i = Q_1 + Q_0 \tag{4}$$

is the plume volume flux at z_i. For $z > z_i$, $w = 0$ and $\Delta = \Delta_2$, a constant value. At $z = z_i$, the buoyancy exhibits a discontinuity from zero to Δ_2, but the interface thickness would be finite because of diffusion of the property producing buoyancy. The environment of the plume consists of the two regions sketched in figure 1, with the entrainment for the region below the interface supplied by the recirculation from the bottom of the container like that for a plume impinging on a solid wall. Conservation of buoyancy flux requires that Δ_2 be the mean buoyancy in the plume at $z = z_i$:

$$\Delta_2 = \frac{F_0}{Q_i} = \frac{Q_0}{Q_0+Q_1}\Delta_0, \tag{5}$$

i.e. a value predictable from the imposed conditions.

This simple derivation gives no indication whether an interface does indeed exist, so a more general approach is required. A dimensional analysis of all variables controlling the elevation and thickness δ of the interface based on the fluxes from the source produced the following set of parameters:

$$\frac{zM_0^{\frac{1}{2}}}{Q_0}, \quad \frac{\delta M_0^{\frac{1}{2}}}{Q_0} = f\left(\frac{Q_1^{\frac{3}{5}}M_0^{\frac{1}{2}}}{Q_0 F_0^{\frac{1}{5}}}, \frac{M_0^{\frac{5}{2}}}{Q_0^2 F_0}, \frac{AM_0}{Q_0^2}, \frac{HM_0^{\frac{1}{2}}}{Q_0}, \frac{M_0^{\frac{1}{2}}}{\kappa}\right) \tag{6}$$

The first term on the right-hand side is the form used for volume flux for a pure plume. Baines & Turner (1969) show that, for a constant entrainment coefficient α, Q is given by

$$\frac{Q^{\frac{3}{5}}}{F_0^{\frac{1}{5}}} = 3^{\frac{1}{5}}(\tfrac{6}{5}\alpha)^{\frac{4}{5}}\pi^{\frac{2}{5}}z, \tag{7}$$

so the dimensionless parameters are obtained by dividing each side by the source lengthscale $Q_0/M_0^{\frac{1}{2}}$. The second term is the square of the Froude number Fr_0 at the source. It is the ratio of the inertia and buoyancy of the source flow: the Froude number of a pure plume is constant along the length at the value

$$Fr^2 = \frac{M^{\frac{5}{2}}}{Q^2F_0} = \frac{5\sqrt{2}}{8\pi^{\frac{1}{2}}\alpha}. \tag{8}$$

At the source Fr_0^2 must be four times this value because the velocity and density have top-hat profiles. The momentum of a forced plume is larger than that of a pure plume with the same buoyancy flux. Thus the Froude number increases if the forced plume becomes more jetlike. For a pure jet, the Froude number is infinite.

The third term, the ratio of the area of tank to area of source, should not influence the flow if it is large. However, as the area is reduced, the inertia of the flow in the environment is increased and this should tend to prevent the interface formation. Similar comments apply to the fourth term, which defines the relative length of the containing walls. Increasing the length increases the upward volume flux in the environment below the interface.

The last term is the Péclet number of the flow. Observations of plumes on the laboratory scale lead to the conclusion that the diffusivity of the buoyancy should affect only the thickness of the interface.

3. Experimental demonstrations

A tank was constructed with vertical glass walls 51 cm high and 32 cm square cross-section. The open bottom was connected to a contraction 56 cm long leading to a pipe 2.54 cm diameter in which a flowmeter was installed. A long weir was constructed along the top to reduce the disturbances on the interface. Fresh water

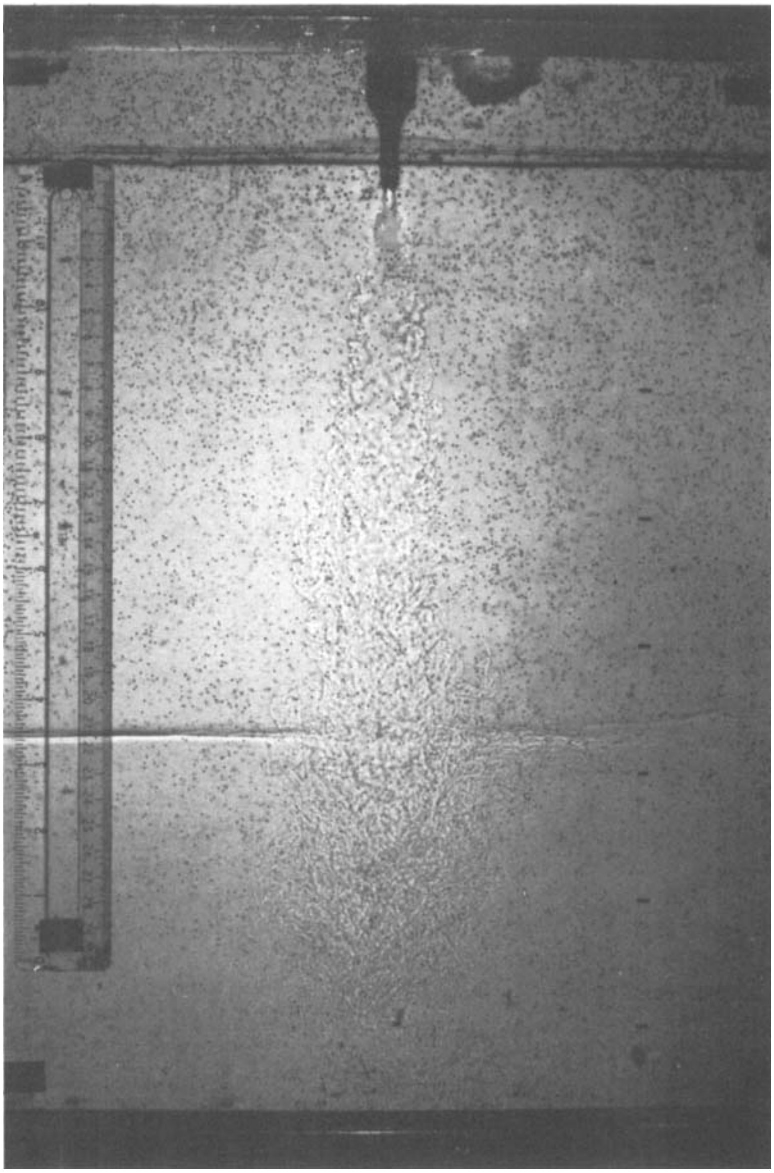

FIGURE 2. Shadowgraph of a forced plume, $Q_0 = 7.9$ cm^3/s, $\Delta_0 = 0.0105$ g, $Q_1 = 127.5$ cm^3/s.

was introduced through a manifold distributor at the top at a rate sufficient to give a very small head over the weir. Plumes were produced by introducing salt through a source 0.52 cm diameter with a screen installed inside to ensure turbulent flow. A second flowmeter was installed in the source supply.

A period of time elapsed after starting before an interface appeared and established at a fixed elevation. This time was about that given by dividing the volume of the tank by Q_0. During an experiment Q_0 was held constant and Q_1 varied. The time required for the interface to move to a new stable level after a change in Q_1 was small compared to the time to establish it.

The shadowgraph technique was used to illuminate the interface and the plume. Figures 2 and 3 are photographs of typical cases. The interface is clearly visible even

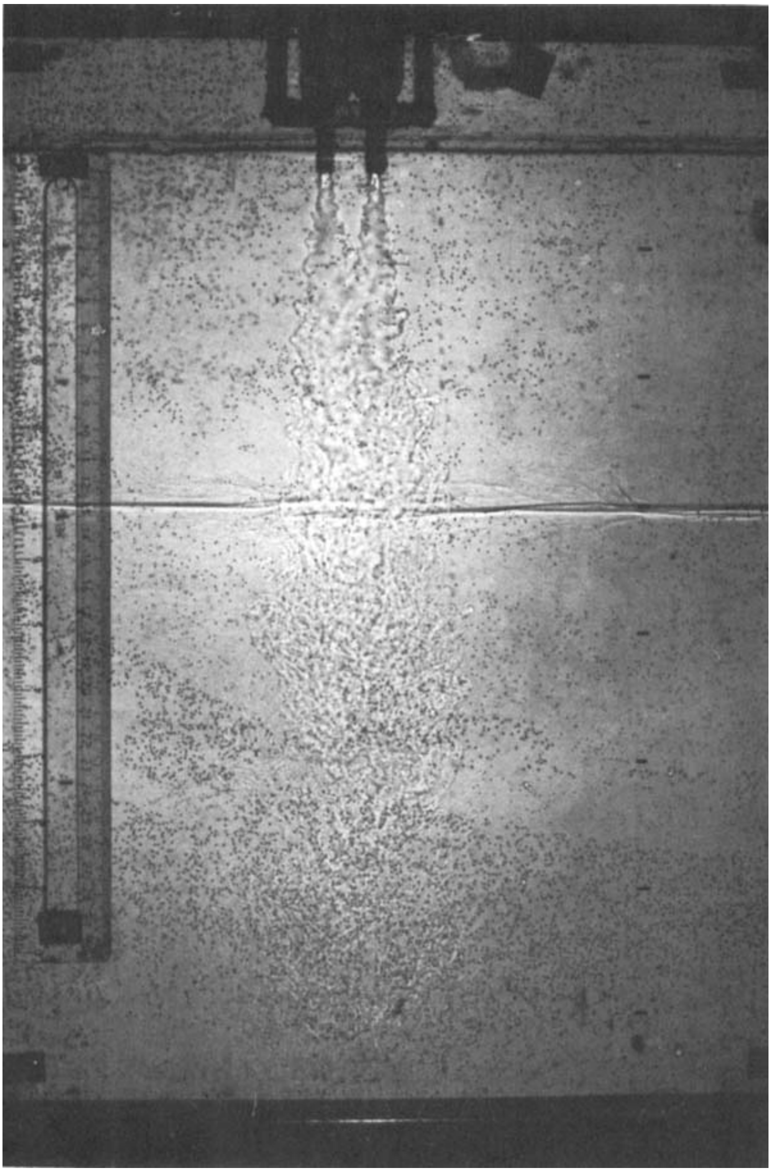

FIGURE 3. Shadowgraph of a pair of forced plumes, $Q_0 = 3.95$ cm^2/s for each source, $\Delta_0 = 0.009$ g, $Q_1 = 81.6$ cm^3/s.

though the relative density difference in one case was only 7.5×10^{-4}. The recirculation pattern below the interface is not visible because the mean density difference is zero. However, at and just below the interface, the plume is visible because of the turbulent density differences carried across it.

Measured interface elevations are presented on figure 4 for a series of experiments covering a range of source density difference. The form of horizontal coordinate was based on (7) so that a straight line would result for a pure plume. The slope of the line produced by the 18 % salt solution when inserted in (7) yields a value of 0.074 for α. This is less than the average value of 0.093 reported in the literature but is not outside the range reported. It also defines the Froude number for a pure plume

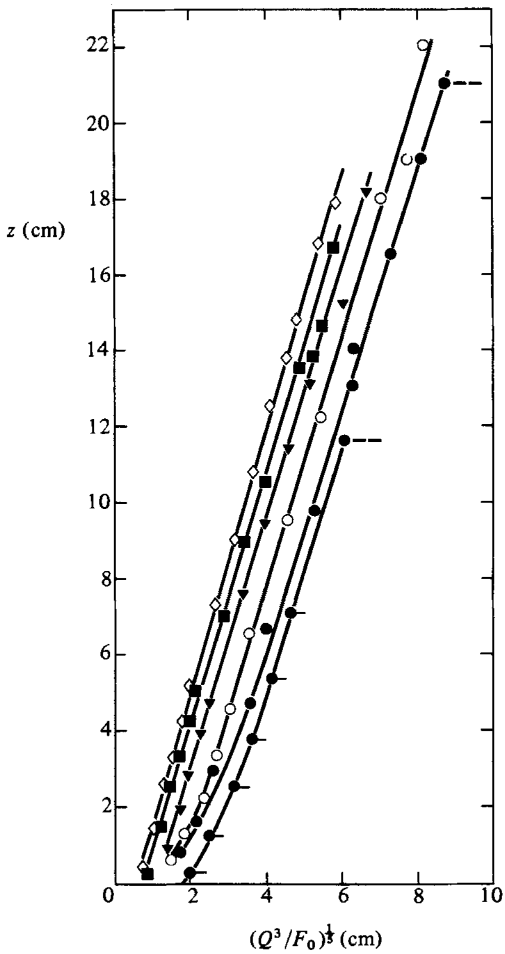

FIGURE 4. Interface level as function of volume flux for a plume: ●-, $\Delta_0 = 0.0025$ g, $Fr_0^2 = 1218$; ●, 0.005 g, 609; ⊙, 0.01 g, 304; ▼, 0.05 g, 52.4; ■, 0.10 g, 22.5; ◇, 0.18 g, 0.87; ---, limiting elevation for existence of an interface.

by (8) as $Fr^2 = 6.74$. The source Froude number for both this case and the 10% density difference are small enough to produce a pure plume at the end of the establishment region. Thus the two curves at the left should be straight lines. The other four cases are forced plumes at the source and approach pure plumes as the elevation increases. Jetlike variation of Q gives a variation of slope which increases with z until the slope of a pure plume is reached. The three curves on the right show this curvature. The smaller the density difference the longer the distance between the source and the level where $Q^{\frac{3}{5}}$ becomes proportional to z. However, this distance is much smaller than that defined by Chu, Senior & List (1981) from measurement of density on the plume centreline. It may be that the discharge of a forced plume varies like that in a pure plume well upstream of the elevation where local variables display the pure plume characteristics.

The interface became less sharp as the density difference was reduced, but it could always be discerned if it existed. In two cases the interface did not exist beyond the elevations noted on figure 4. The reason why no interface formed was deduced from observing the motion in the environment. The large eddies which moved upward with the entrainment flow had sufficient inertia to move well above the level where an

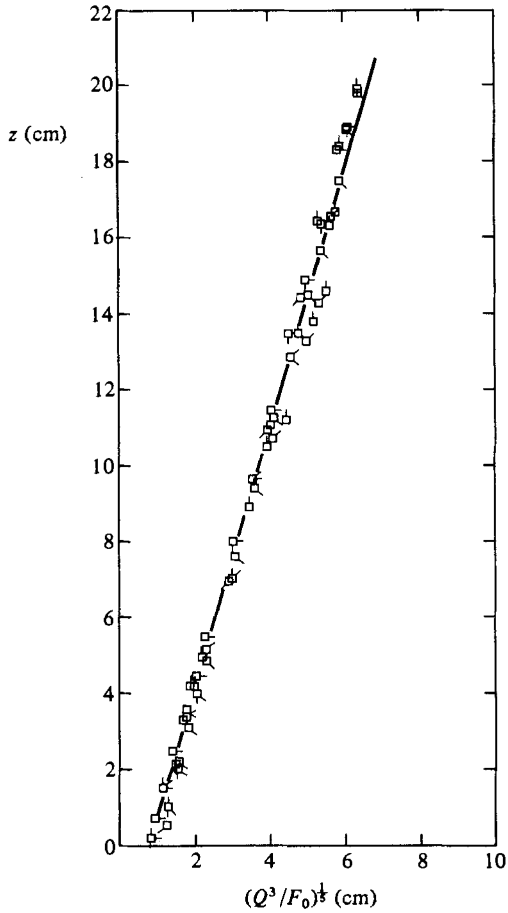

FIGURE 5. Interface level as function of volume flux for different environment cross-section: $\Delta_0 = 0.10$ g, $Fr_0^2 = 22.5$; ⊡, 32.1 cm square; ⊡, 25.7 cm square; ⊡ 22.9 cm square; ⊡, 19.1 cm square; ⊡, 15.2 cm square; ⊡, 10.2 cm square.

interface would exist. These eddies mixed with the downward entrainment flow and so reduced the effective density difference. It is concluded that H is the important factor as well as the geometry of the field beyond the end of the walls. If the walls had been longer and if a larger volume has existed below the walls the interface would have been found at larger values of z. In other words, the existence of an interface is a function of the second and fourth terms in (6).

The effect of the cross-sectional area of the environment was investigated by constructing a series of 5 square tubes which were inserted in the tank. The same source and discharge were used as in the discussion above and the cross-sectional area was reduced progressively until changes in the flow and interface were observed. It was anticipated that two effects would be seen because of the increase in the vertical environment velocity. First, the entrainment rate should be reduced because the relative velocity is reduced. Secondly, the inertia of the environment should affect the establishment of the interface. However, neither of these effects was strong enough to be measured. The data plotted in figure 5 scatter about the line on figure 4, showing that any reduction of Q as the environment velocity increases is less than the accuracy of the experiment. An interface was seen at all levels above that where the visible edge of the plume touched the wall. As the plume expands to the width of the tank,

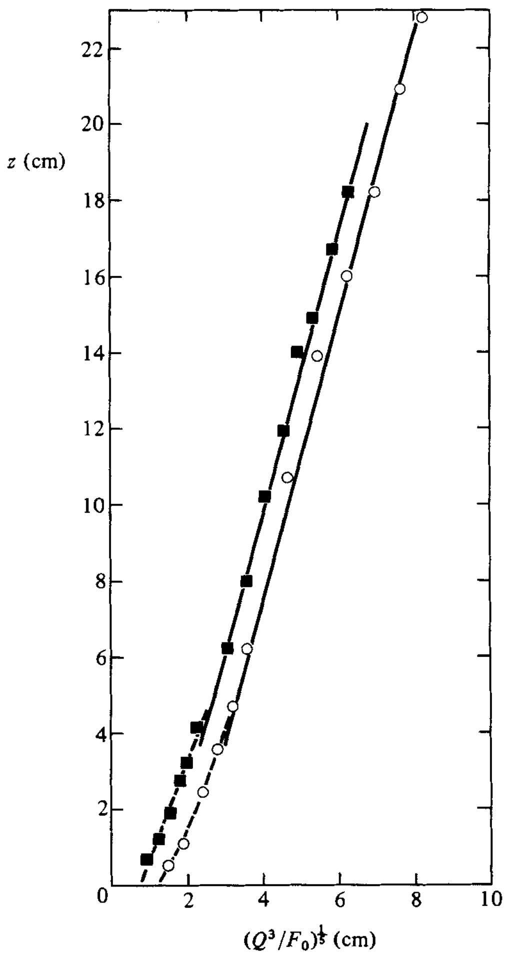

FIGURE 6. Interface level as function of volume flux for a pair of plumes, $Q_0 = 3.95$ cm^3/s for each source: ⊙, $\Delta_0 = 0.01$ g, ■, $\Delta_0 = 0.10$ g; ——, lines of slope equal to corresponding case on figure 4; – – –, lines for plumes entraining independently.

it occupies the entire cross-section and no environment exists. There was, however, an influence of the small cross-section when the interface was close to the elevation of the source. The strong upward flow in the environment appeared to induce random oscillations of the plume, and it flowed along one of the walls for period of a few seconds. The oscillation produced waves on the interface which reduced the accuracy of determination of elevation.

The thickness of the interface did not appear to change with time after establishment, but it did vary with elevation during an experiment. It was not possible to make accurate measurements of the thickness from the shadowgraph, but it was noticeably thinner closer to the source. However, the thickness can be analysed approximately by assuming the interface to be a region of one-dimensional diffusion. The mass-conservation equation

$$\frac{\partial}{\partial z} wc = \kappa \frac{\partial^2 c}{\partial z^2} \tag{9}$$

is readily integrated assuming that w varies linearly with distance from the centre of the interface. At the centre

$$c_0 = \tfrac{1}{2}(c_1 + c_2), \tag{10}$$

where c_1 and c_2 are the concentrations of the buoyant property in the upper and lower regions. The result is expressed in terms of z measured from the centre of the interface as

$$\frac{c-c_1}{c_0-c_1} = e^{y^2}(1-\operatorname{erf} y), \tag{11}$$

where

$$y = -\left(\frac{q_e}{2A\kappa}\right)^{\frac{1}{2}} z \tag{12}$$

For large y, (11) can be approximated by

$$\frac{c-c_1}{c_0-c_1} = \frac{1}{\pi^{\frac{1}{2}}y}\left(1-\frac{1}{2y^2}+\dots\right), \tag{11a}$$

which yields $y = 2.6$ for a relative concentration of 0.2 or 10% of the total concentration difference. This value of y can be used to calculate the half-thickness z for the experiment shown in figure 2. The result of 2.2 mm is about the thickness of the grey zone seen in the left-hand side of the photograph. The relationship of (12) indicates that the thickness should decrease with increasing q_e – that is, with the distance from the source – and increase both with width of the environment and diffusivity.

Measurements of the volume flux for the plumes issuing from two sources spaced at 2.0 cm are plotted on figure 6. The variations are as anticipated. Close to the source, the plumes should entrain fluid independently from the environment. Thus, for two sources with the same total volume and buoyancy fluxes as a single source, the value of $(Q^3/F_0)^{\frac{1}{5}}$ should be increased by the factor $2^{\frac{2}{5}}$. The dashed lines on figure 5 are the lines on figure 4 multiplied by 1.32, and it is evident that the fit with the data is good. Far downstream of the sources, the plumes become a single flow which should have the characteristics of a plume from a single source. The flow is similar to the multiple jets studied by Baines & Keffer (1975). On the shadowgraphs, the width was seen to be roughly constant in the region between 5 and 20 cm from the source and beyond 20 cm to expand like a single-source plume. In the direction normal to the photo, the width expanded continuously so the plume cross-sectional area increases with z. On figure 6, the slopes of the volume flux line for single-source plumes are plotted for $z > 5$ cm. It is seen that the agreement is very good in both cases. The change from entrainment as separate plumes to that for a single plume occurs over a distance much shorter than a plume diameter. The plume cross-section changes to circular over a much longer distance.

4. An industrial application

In many large buildings in the metallurgical industries there are open heated surfaces at floor level. Plumes are formed which rise towards the roof and usually contain pollutants. Kettles of molten lead in a lead refinery and electrolytic cells in a zinc or copper refinery are examples. In most installations constructed early in this century, the building was ventilated by openings in the roof and in the walls. As a result, pollution was dispersed over the surrounding countryside, and for particular wind directions workers inside were subjected to high temperature and pollution levels.

The concept of establishing an interface in the building has been applied in the design of ventilation systems for magnesium, nickel and zinc refineries. The building was sealed except for openings in the floor and the walls at floor level as sketched

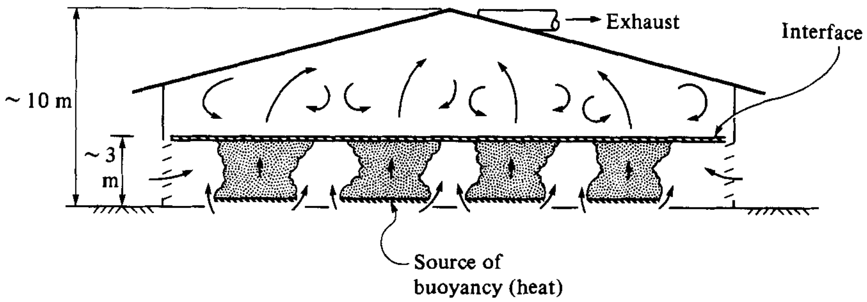

FIGURE 7. Sketch of flow in a metallurgical cell house.

on figure 7. Just sufficient air is exhausted to establish an interface about 3 m above the floor. This requires the exhausting of a volume flux equal to the total entrainment to the lower 3 m of all plumes in the building. The result is a relatively cool, pollution-free zone at floor level. Furthermore, the filters or other cleaning devices are required to handle a minimum volume flux. This represents a saving in capital and operating costs compared with a conventional design wherein air is introduced at all levels and all of the flux entrained by the plumes is filtered or scrubbed.

The assistance of the Natural Sciences and Engineering Research Council of Canada is acknowledged for the support of the construction and testing of the apparatus under Grant A-1066. Most of the tests were performed by Mr E. Morala, whose care is acknowledged.

REFERENCES

BAINES, W. D. & KEFFER, J. F. 1975 Entrainment by a multiple source turbulent jet. *Adv. Geophys.* **18b**, 189–198.

BAINES, W. D. & TURNER, J. S. 1969 Turbulent buoyant convection from a source in a confined region. *J. Fluid Mech.* **37**, 51–80.

CHU, V. H., SENIOR, C. & LIST, J. 1981 *ASME Paper* 81-FE-29.

RICOU, F. P. & SPALDING, D. B. 1961 Measurements of entrainment by axisymmetrical turbulent jets. *J. Fluid Mech.* **11**, 21–32.